# Package 'DiscreteFWER'

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	FWER-Based Multiple Testing Procedures with Adaptation for Discrete Tests
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Date 2	2024-11-26
t 1 2 2 0 0 4 1 1 0 0 0 1	<b>ption</b> Implementations of several multiple testing procedures that control the family-wise error rate (FWER) designed specifically for discrete tests. Included are discrete adaptations of the Bonferroni, Holm, Hochberg and Šidák procedures as described in the papers Döhler (2010) ``Validation of credit default probabilities using multiple-testing procedures'' <doi:10.21314 jrmv.2010.062=""> and Zhu &amp; Guo (2019) ``Family-Wise Error Rate Controlling Procedures for Discrete Data'' <doi:10.1080 19466315.2019.1654912="">. The main procedures of this package take as input the results of a test procedure from package 'DiscreteTests' or a set of observed p-values and their discrete support under their nulls. A shortcut function to apply discrete procedures directly to data is also provided.</doi:10.1080></doi:10.21314>
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Autho	r Sebastian Döhler [aut, ctb] ( <https: 0000-0002-0321-6355="" orcid.org="">), Florian Junge [aut, ctb, cre] (<https: 0009-0001-6856-6938="" orcid.org="">)</https:></https:>

Maintainer Florian Junge <diso.fbmn@h-da.de> Repository CRAN Date/Publication 2024-11-27 13:00:05 UTC

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DiscreteFWER-package FWER-Based Multiple Testing Procedures with Adaptation for Discrete Tests

#### Description

This package implements adaptions for discrete tests of the Bonferroni, Holm, Hochberg and Šidák procedures for control of the family-wise error rate (FWER).

#### Details

The main function discrete\_FWER() makes all four procedures available to the user. DBonferroni(), DHolm(), DHochberg() and DSidak() are wrapper functions that enable the user to access them directly. Their main parameters are either a DiscreteTestResults object from package DiscreteTests or a vector of raw observed p-values and a list whose elements are the discrete supports of the CDFs of the *p*-values.

The function direct\_discrete\_FWER() is a wrapper for DiscreteFDR::generate.pvalues() and discrete\_FWER(), which applies discrete procedures directly to data.

#### Author(s)

Maintainer: Florian Junge <diso.fbmn@h-da.de>(ORCID) [contributor]

Authors:

• Sebastian Döhler <sebastian.doehler@h-da.de> (ORCID) [contributor]

### DBonferroni

#### References

Döhler, S. (2010). Validation of credit default probabilities using multiple-testing procedures. *Journal of Risk Model Validation*, 4(4), 59-92. doi:10.21314/JRMV.2010.062

Zhu, Y., & Guo, W. (2019). Family-Wise Error Rate Controlling Procedures for Discrete Data. *Statistics in Biopharmaceutical Research*, *12*(1), 117-128. doi:10.1080/19466315.2019.1654912

### See Also

Useful links:

- https://github.com/DISOhda/DiscreteFWER
- Report bugs at https://github.com/DISOhda/DiscreteFWER/issues

DBonferroni

#### Discrete Bonferroni Procedure

### Description

DBonferroni() is a wrapper function of discrete\_FWER() for computing the discrete Bonferroni procedure for tests with an arbitrary dependency structure. It simply passes its arguments to discrete\_FWER() with fixed independence = FALSE and single\_step = TRUE.

### Usage

```
DBonferroni(test_results, ...)
## Default S3 method:
DBonferroni(
  test_results,
  pCDFlist,
  alpha = 0.05,
  critical_values = FALSE,
  select_threshold = 1,
  pCDFlist_indices = NULL,
)
## S3 method for class 'DiscreteTestResults'
DBonferroni(
  test_results,
  alpha = 0.05,
  critical_values = FALSE,
  select_threshold = 1,
  . . .
)
```

### Arguments

test_results	either a numeric vector with <i>p</i> -values or an R6 object of class DiscreteTestResults from package DiscreteTests for which a discrete FWER procedure is to be performed.	
	further arguments to be passed to or from other methods. They are ignored here.	
pCDFlist	list of the supports of the CDFs of the <i>p</i> -values; each list item must be a numeric vector, which is sorted in increasing order and whose last element equals 1.	
alpha	single real number strictly between 0 and 1 indicating the target FWER level.	
critical_values	6	
	single boolean specifying whether critical constants are to be computed.	
select_threshold		
	single real number strictly between 0 and 1 indicating the largest raw <i>p</i> -value to be considered, i.e. only <i>p</i> -values below this threshold are considered and the procedures are adjusted in order to take this selection effect into account; if select_threshold = 1 (the default), all raw <i>p</i> -values are selected.	
pCDFlist_indices		
	list of numeric vectors containing the test indices that indicate to which raw <i>p</i> -value(s) each support in pCDFlist belongs; if NULL (the default) the lengths of test_results and pCDFlist <b>must</b> be equal.	

#### Details

Computing critical constants (critical\_values = TRUE) requires considerably more execution time, especially if the number of unique supports is large. We recommend that users should only have them calculated when they need them, e.g. for illustrating the rejection set in a plot or other theoretical reasons. Setting (critical\_values = FALSE) is sufficient for obtaining rejection decisions and adjusted *p*-values.

### Value

A DiscreteFWER S3 class object whose elements are:

Rejected	rejected raw <i>p</i> -values.	
Indices	indices of rejected hypotheses.	
Num_rejected	number of rejections.	
Adjusted	adjusted p-values.	
Critical_consta	ants	
	critical values (only exists if computations where performed with critical_values = TRUE).	
Data	list with input data.	
Data\$Method	character string describing the performed algorithm, e.g. 'Discrete Bonferroni procedure'.	
Data\$Raw_pvalues		
	observed <i>p</i> -values.	
Data\$pCDFlist	list of the <i>p</i> -value supports.	

### DBonferroni

Data\$FWER_level		
	FWER level alpha.	
Data\$Independer	nce	
	boolean indicating whether the <i>p</i> -values were considered as independent.	
Data\$Single_ste	2p	
	boolean indicating whether a single-step or step-down procedure was performed.	
Data\$Data_name	the respective variable names of the input data.	
Select	list with data related to <i>p</i> -value selection; only exists if select_threshold < 1.	
Select\$Threshold		
	<i>p</i> -value selection threshold (select_threshold).	
Select\$Effective_Thresholds		
	results of each <i>p</i> -value CDF evaluated at the selection threshold.	
Select\$Pvalues	selected <i>p</i> -values that are $\leq$ selection threshold.	
Select\$Indices	indices of $p$ -values $\leq$ selection threshold.	
Select\$Scaled	scaled selected <i>p</i> -values.	
Select\$Number	number of selected $p$ -values $\leq$ selection threshold.	

#### References

Döhler, S. (2010). Validation of credit default probabilities using multiple-testing procedures. *Journal of Risk Model Validation*, 4(4), 59-92. doi:10.21314/JRMV.2010.062

Zhu, Y., & Guo, W. (2019). Family-Wise Error Rate Controlling Procedures for Discrete Data. *Statistics in Biopharmaceutical Research*, *12*(1), 117-128. doi:10.1080/19466315.2019.1654912

### See Also

discrete\_FWER(), DHolm(), DSidak(), DHochberg()

### Examples

summary(DBonferroni\_fast)

```
X1 <- c(4, 2, 2, 14, 6, 9, 4, 0, 1)
X2 <- c(0, 0, 1, 3, 2, 1, 2, 2, 2)
N1 <- rep(148, 9)
N2 <- rep(132, 9)
Y1 <- N1 - X1
Y2 <- N2 - X2
df <- data.frame(X1, Y1, X2, Y2)
df
# Computation of p-values and their supports with Fisher's exact test
library(DiscreteTests) # for Fisher's exact test
test_results <- fisher_test_pv(df)
raw_pvalues <- test_results$get_pvalues()
pCDFlist <- test_results$get_pvalue_supports()
# d-Bonferroni without critical values; using extracted p-values and supports
DBonferroni_fast <- DBonferroni(raw_pvalues, pCDFlist)</pre>
```

```
# d-Bonferroni with critical values; using test results object
DBonferroni_crit <- DBonferroni(test_results, critical_values = TRUE)
summary(DBonferroni_crit)
```

DHochberg

Discrete Hochberg Procedure

### Description

DHochberg() is a wrapper function of discrete\_FWER() for computing the discrete Hochberg stepup procedure for independent or positively correlated discrete tests. It simply passes its arguments to discrete\_FWER() with fixed independence = TRUE and single\_step = FALSE.

#### Usage

```
DHochberg(test_results, ...)
## Default S3 method:
DHochberg(
  test_results,
  pCDFlist,
  alpha = 0.05,
  critical_values = FALSE,
  select_threshold = 1,
  pCDFlist_indices = NULL,
  . . .
)
## S3 method for class 'DiscreteTestResults'
DHochberg(
  test_results,
  alpha = 0.05,
 critical_values = FALSE,
  select_threshold = 1,
  . . .
)
```

### Arguments

test_results	either a numeric vector with <i>p</i> -values or an R6 object of class DiscreteTestResults from package DiscreteTests for which a discrete FWER procedure is to be performed.
	further arguments to be passed to or from other methods. They are ignored here.
pCDFlist	list of the supports of the CDFs of the <i>p</i> -values; each list item must be a numeric vector, which is sorted in increasing order and whose last element equals 1.

### DHochberg

alpha	single real number strictly between 0 and 1 indicating the target FWER level.	
critical_values		
	single boolean specifying whether critical constants are to be computed.	
select_threshol	d	
	single real number strictly between 0 and 1 indicating the largest raw <i>p</i> -value to be considered, i.e. only <i>p</i> -values below this threshold are considered and the procedures are adjusted in order to take this selection effect into account; if select_threshold = 1 (the default), all raw <i>p</i> -values are selected.	
pCDFlist_indices		
	list of numeric vectors containing the test indices that indicate to which raw $p$ -value(s) each support in pCDFlist belongs; if NULL (the default) the lengths of test_results and pCDFlist <b>must</b> be equal.	

### Details

Computing critical constants (critical\_values = TRUE) requires considerably more execution time, especially if the number of unique supports is large. We recommend that users should only have them calculated when they need them, e.g. for illustrating the rejection set in a plot or other theoretical reasons. Setting (critical\_values = FALSE) is sufficient for obtaining rejection decisions and adjusted *p*-values.

### Value

A DiscreteFWER S3 class object whose elements are:

Rejected	rejected raw <i>p</i> -values.	
Indices	indices of rejected hypotheses.	
Num_rejected	number of rejections.	
Adjusted	adjusted <i>p</i> -values.	
Critical_consta	nts	
	critical values (only exists if computations where performed with critical_values = TRUE).	
Data	list with input data.	
Data\$Method	character string describing the performed algorithm, e.g. 'Discrete Bonferroni procedure'.	
Data\$Raw_pvalues		
	observed <i>p</i> -values.	
Data\$pCDFlist	list of the <i>p</i> -value supports.	
Data\$FWER_level		
	FWER level alpha.	
Data\$Independence		
	boolean indicating whether the <i>p</i> -values were considered as independent.	
Data\$Single_step		
	boolean indicating whether a single-step or step-down procedure was performed.	
Data\$Data_name	the respective variable names of the input data.	

Select	list with data related to <i>p</i> -value selection; only exists if select_threshold < 1.	
Select\$Threshold		
	$p$ -value selection threshold (select_threshold).	
Select\$Effective_Thresholds		
	results of each <i>p</i> -value CDF evaluated at the selection threshold.	
Select\$Pvalues	selected <i>p</i> -values that are $\leq$ selection threshold.	
Select\$Indices	indices of $p$ -values $\leq$ selection threshold.	
Select\$Scaled	scaled selected <i>p</i> -values.	
Select\$Number	number of selected $p$ -values $\leq$ selection threshold.	

#### References

Zhu, Y., & Guo, W. (2019). Family-Wise Error Rate Controlling Procedures for Discrete Data. *Statistics in Biopharmaceutical Research*, *12*(1), 117-128. doi:10.1080/19466315.2019.1654912

#### See Also

discrete\_FWER(), DSidak(), DBonferroni(), DHolm

### Examples

```
X1 <- c(4, 2, 2, 14, 6, 9, 4, 0, 1)
X2 <- c(0, 0, 1, 3, 2, 1, 2, 2, 2)
N1 <- rep(148, 9)
N2 <- rep(132, 9)
Y1 <- N1 - X1
Y2 <- N2 - X2
df <- data.frame(X1, Y1, X2, Y2)
df
# Computation of p-values and their supports with Fisher's exact test
library(DiscreteTests) # for Fisher's exact test
test_results <- fisher_test_pv(df)
raw_pvalues <- test_results$get_pvalues()
pCDFlist <- test_results$get_pvalue_supports()</pre>
```

```
# d-Hochberg without critical values; using test results object
DHoch_fast <- DHochberg(test_results)
summary(DHoch_fast)
```

```
# d-Hochberg with critical values; using extracted p-values and supports
DHoch_crit <- DHochberg(raw_pvalues, pCDFlist, critical_values = TRUE)
summary(DHoch_crit)
```

DHolm

### Description

DHolm() is a wrapper function of discrete\_FWER() for computing the discrete Holm step-down procedure for tests with an arbitrary dependency structure. It simply passes its arguments to discrete\_FWER() with fixed independence = FALSE and single\_step = FALSE.

#### Usage

```
DHolm(test_results, ...)
## Default S3 method:
DHolm(
  test_results,
  pCDFlist,
  alpha = 0.05,
  critical_values = FALSE,
  select_threshold = 1,
  pCDFlist_indices = NULL,
  . . .
)
## S3 method for class 'DiscreteTestResults'
DHolm(
  test_results,
  alpha = 0.05,
  critical_values = FALSE,
  select_threshold = 1,
  . . .
)
```

### Arguments

test_results	either a numeric vector with <i>p</i> -values or an R6 object of class DiscreteTestResults from package DiscreteTests for which a discrete FWER procedure is to be performed.	
	further arguments to be passed to or from other methods. They are ignored here.	
pCDFlist	list of the supports of the CDFs of the <i>p</i> -values; each list item must be a numeric vector, which is sorted in increasing order and whose last element equals 1.	
alpha	single real number strictly between 0 and 1 indicating the target FWER level.	
critical_values		
	single boolean specifying whether critical constants are to be computed.	

#### select\_threshold

single real number strictly between 0 and 1 indicating the largest raw p-value to be considered, i.e. only p-values below this threshold are considered and the procedures are adjusted in order to take this selection effect into account; if select\_threshold = 1 (the default), all raw p-values are selected.

#### pCDFlist\_indices

list of numeric vectors containing the test indices that indicate to which raw *p*-value(s) each support in pCDFlist belongs; if NULL (the default) the lengths of test\_results and pCDFlist **must** be equal.

### Details

Computing critical constants (critical\_values = TRUE) requires considerably more execution time, especially if the number of unique supports is large. We recommend that users should only have them calculated when they need them, e.g. for illustrating the rejection set in a plot or other theoretical reasons. Setting (critical\_values = FALSE) is sufficient for obtaining rejection decisions and adjusted *p*-values.

#### Value

A DiscreteFWER S3 class object whose elements are:

Rejected	rejected raw <i>p</i> -values.
Indices	indices of rejected hypotheses.
Num_rejected	number of rejections.
Adjusted	adjusted p-values.
Critical_consta	ants
	critical values (only exists if computations where performed with critical_values = TRUE).
Data	list with input data.
Data\$Method	character string describing the performed algorithm, e.g. 'Discrete Bonferroni procedure'.
Data\$Raw_pvalue	25
	observed <i>p</i> -values.
Data\$pCDFlist	list of the <i>p</i> -value supports.
Data\$FWER_level	
	FWER level alpha.
Data\$Independer	nce
	boolean indicating whether the <i>p</i> -values were considered as independent.
Data\$Single_ste	2p
	boolean indicating whether a single-step or step-down procedure was performed.
Data\$Data_name	the respective variable names of the input data.
Select	list with data related to <i>p</i> -value selection; only exists if select_threshold < 1.
Select\$Thresho	ld
	<i>p</i> -value selection threshold (select_threshold).

### DHolm

Select\$Effective_Thresholds		
	results of each $p$ -value CDF evaluated at the selection threshold.	
Select\$Pvalues	selected <i>p</i> -values that are $\leq$ selection threshold.	
Select\$Indices	indices of $p$ -values $\leq$ selection threshold.	
Select\$Scaled	scaled selected <i>p</i> -values.	
Select\$Number	number of selected $p$ -values $\leq$ selection threshold.	

### References

Döhler, S. (2010). Validation of credit default probabilities using multiple-testing procedures. *Journal of Risk Model Validation*, 4(4), 59-92. doi:10.21314/JRMV.2010.062

Zhu, Y., & Guo, W. (2019). Family-Wise Error Rate Controlling Procedures for Discrete Data. *Statistics in Biopharmaceutical Research*, *12*(1), 117-128. doi:10.1080/19466315.2019.1654912

### See Also

discrete\_FWER(), DBonferroni(), DSidak(), DHochberg()

#### Examples

```
X1 <- c(4, 2, 2, 14, 6, 9, 4, 0, 1)
X2 <- c(0, 0, 1, 3, 2, 1, 2, 2, 2)
N1 <- rep(148, 9)
N2 <- rep(132, 9)
Y1 <- N1 - X1
Y2 <- N2 - X2
df <- data.frame(X1, Y1, X2, Y2)
df
# Computation of p-values and their supports with Fisher's exact test
library(DiscreteTests) # for Fisher's exact test
test_results <- fisher_test_pv(df)</pre>
raw_pvalues <- test_results$get_pvalues()</pre>
pCDFlist <- test_results$get_pvalue_supports()</pre>
# d-Holm without critical values; using extracted p-values and supports
DHolm_fast <- DHolm(raw_pvalues, pCDFlist)</pre>
summary(DHolm_fast)
# d-Holm with critical values; using test results object
DHolm_crit <- DHolm(test_results, critical_values = TRUE)</pre>
```

```
summary(DHolm_crit)
```

direct\_discrete\_FWER Direct Application of Multiple Testing Procedures to Dataset

### Description

Apply one of the various FWER adaptation procedures, with or without computing the critical constants, to a data set of 2x2 contingency tables using statistical test functions from package DiscreteTests. If necessary, functions for pre-processing can be passed as well.

#### Usage

```
direct_discrete_FWER(
   dat,
   test_fun,
   test_args = NULL,
   alpha = 0.05,
   independence = FALSE,
   single_step = TRUE,
   critical_values = FALSE,
   select_threshold = 1,
   preprocess_fun = NULL,
   preprocess_args = NULL
)
```

#### Arguments

dat	input data; must be suitable for the first parameter of the provided preprocess_fun function or, if preprocess_fun is NULL, for the first parameter of the test_fun function.
test_fun	function <b>from package</b> DiscreteTests, i.e. one whose name ends with *_test_pv and which performs hypothesis tests and provides an object with <i>p</i> -values and their support sets; can be specified by a single character string (which is automatically checked for being a suitable function <b>from that package</b> and may be abbreviated) or a single function object.
test_args	optional named list with arguments for test_fun; the names of the list fields must match the test function's parameter names. The first parameter of the test function (i.e. the data) MUST NOT be included!
alpha	single real number strictly between 0 and 1 indicating the target FWER level.
independence	single boolean specifying whether the <i>p</i> -values are statistically independent or not.
single_step	single boolean specifying whether to perform a single-step (TRUE) or step-down (FALSE; the default) procedure.
critical_values	
	single boolean specifying whether critical constants are to be computed.

select_threshold		
	single real number strictly between 0 and 1 indicating the largest raw <i>p</i> -value to be considered, i.e. only <i>p</i> -values below this threshold are considered and the procedures are adjusted in order to take this selection effect into account; if select_threshold = 1 (the default), all raw <i>p</i> -values are selected.	
preprocess_fun	optional function for pre-processing the input data; its result must be suitable for the first parameter of the test_fun function.	
preprocess_args		
	optional named list with arguments for preprocess_fun; the names of the list fields must match the pre-processing function's parameter names. The first parameter of the test function (i.e. the data) MUST NOT be included!	

### Value

A DiscreteFWER S3 class object whose elements are:

Rejected	rejected raw <i>p</i> -values.	
Indices	indices of rejected hypotheses.	
Num_rejected	number of rejections.	
Adjusted	adjusted p-values.	
Critical_consta	ants	
	critical values (only exists if computations where performed with critical_values = TRUE).	
Data	list with input data.	
Data\$Method	character string describing the performed algorithm, e.g. 'Discrete Bonferroni procedure'.	
Data\$Raw_pvalue	25	
	observed <i>p</i> -values.	
Data\$pCDFlist	list of the <i>p</i> -value supports.	
Data\$FWER_level		
	FWER level alpha.	
Data\$Independen		
Data\$Single_ste	boolean indicating whether the $p$ -values were considered as independent.	
Data#Jingit_Jte	boolean indicating whether a single-step or step-down procedure was performed.	
Data\$Data_name	the respective variable names of the input data.	
Select	list with data related to $p$ -value selection; only exists if select_threshold < 1.	
Select\$Threshold		
	$p$ -value selection threshold (select_threshold).	
Select\$Effective_Thresholds		
	results of each <i>p</i> -value CDF evaluated at the selection threshold.	
Select\$Pvalues	selected <i>p</i> -values that are $\leq$ selection threshold.	
Select\$Indices	indices of $p$ -values $\leq$ selection threshold.	
Select\$Scaled	scaled selected <i>p</i> -values.	
Select\$Number	number of selected $p$ -values $\leq$ selection threshold.	

#### Examples

```
X1 <- c(4, 2, 2, 14, 6, 9, 4, 0, 1)
X2 <- c(0, 0, 1, 3, 2, 1, 2, 2, 2)
N1 <- rep(148, 9)
N2 <- rep(132, 9)
Y1 <- N1 - X1
Y2 <- N2 - X2
df <- data.frame(X1, Y1, X2, Y2)
df
# Computation of p-values and their supports with Fisher's exact test
library(DiscreteTests) # for Fisher's exact test
test_results <- fisher_test_pv(df)</pre>
raw_pvalues <- test_results$get_pvalues()</pre>
pCDFlist <- test_results$get_pvalue_supports()</pre>
DBonf <- direct_discrete_FWER(df, "fisher")</pre>
summary(DBonf)
DHolm <- direct_discrete_FWER(df, "fisher_test_pv", single_step = FALSE)</pre>
summary(DHolm)
DBonf_bin <- direct_discrete_FWER(X1 + X2, "binom_test_pv",</pre>
                                    list(n = N1 + N2, p = 0.05))
summary(DBonf_bin)
DHolm_bin <- direct_discrete_FWER(X1 + X2, "binom",</pre>
                                    list(n = N1 + N2, p = 0.05),
                                    single_step = TRUE)
summary(DHolm_bin)
```

discrete\_FWER Discrete Family-wise Error Rate (FWER) Adaptation Procedures

#### Description

Apply a discrete FWER adaptation procedure, with or without computing the critical values, to a set of p-values and their discrete support.

#### Usage

```
discrete_FWER(test_results, ...)
## Default S3 method:
discrete_FWER(
   test_results,
   pCDFlist,
   alpha = 0.05,
```

```
independence = FALSE,
  single_step = FALSE,
  critical_values = FALSE,
  select_threshold = 1,
  pCDFlist_indices = NULL,
  . . .
)
## S3 method for class 'DiscreteTestResults'
discrete_FWER(
  test_results,
  alpha = 0.05,
  independence = FALSE,
  single_step = FALSE,
  critical_values = FALSE,
  select_threshold = 1,
  . . .
)
```

#### Arguments

	either a numeric vector with <i>p</i> -values or an R6 object of class DiscreteTestResults from package DiscreteTests for which a discrete FWER procedure is to be performed.
	further arguments to be passed to or from other methods. They are ignored here.
	list of the supports of the CDFs of the <i>p</i> -values; each list item must be a numeric vector, which is sorted in increasing order and whose last element equals 1.
alpha	single real number strictly between 0 and 1 indicating the target FWER level.
	single boolean specifying whether the <i>p</i> -values are statistically independent or not.
single_step	single boolean specifying whether to perform a single-step (TRUE) or step-down (FALSE; the default) procedure.
critical_values	
	single boolean specifying whether critical constants are to be computed.
select_threshol	d
	single real number strictly between 0 and 1 indicating the largest raw <i>p</i> -value to be considered, i.e. only <i>p</i> -values below this threshold are considered and the procedures are adjusted in order to take this selection effect into account; if select_threshold = 1 (the default), all raw <i>p</i> -values are selected.
pCDFlist_indice	S
	list of numeric vectors containing the test indices that indicate to which raw <i>p</i> -value(s) each support in pCDFlist belongs; if NULL (the default) the lengths of test_results and pCDFlist <b>must</b> be equal.

### Details

Computing critical constants (critical\_values = TRUE) requires considerably more execution time, especially if the number of unique supports is large. We recommend that users should only have

them calculated when they need them, e.g. for illustrating the rejection set in a plot or other theoretical reasons. Setting (critical\_values = FALSE) is sufficient for obtaining rejection decisions and adjusted *p*-values.

Depending on the choices of independence and single\_step, one of the following procedures, is applied:

	single-step	stepwise
independent	Šidák	Hochberg (step-up)
not independent	Bonferroni	Holm (step-down)

Each procedure is available by its own shortcut function:

	single-step	stepwise
independent	DSidak()	DHochberg()
not independent	DBonferroni()	DHolm()

### Value

A DiscreteFWER S3 class object whose elements are:

Rejected	rejected raw <i>p</i> -values.	
Indices	indices of rejected hypotheses.	
Num_rejected	number of rejections.	
Adjusted	adjusted p-values.	
Critical_consta	nts	
	critical values (only exists if computations where performed with critical_values = TRUE).	
Data	list with input data.	
Data\$Method	character string describing the performed algorithm, e.g. 'Discrete Bonferroni procedure'.	
Data\$Raw_pvalues		
	observed <i>p</i> -values.	
Data\$pCDFlist	list of the <i>p</i> -value supports.	
Data\$FWER_level		
	FWER level alpha.	
Data\$Independer	ice	
	boolean indicating whether the <i>p</i> -values were considered as independent.	
Data\$Single_step		
	boolean indicating whether a single-step or step-down procedure was performed.	
Data\$Data_name	the respective variable names of the input data.	
Select	list with data related to <i>p</i> -value selection; only exists if select_threshold < 1.	
Select\$Threshold		
	<i>p</i> -value selection threshold (select_threshold).	

Select\$Effective_Thresholds		
	results of each <i>p</i> -value CDF evaluated at the selection threshold.	
Select\$Pvalues	selected <i>p</i> -values that are $\leq$ selection threshold.	
Select\$Indices	indices of $p$ -values $\leq$ selection threshold.	
Select\$Scaled	scaled selected <i>p</i> -values.	
Select\$Number	number of selected $p$ -values $\leq$ selection threshold.	

### References

Döhler, S. (2010). Validation of credit default probabilities using multiple-testing procedures. *Journal of Risk Model Validation*, 4(4), 59-92. doi:10.21314/JRMV.2010.062

Zhu, Y., & Guo, W. (2019). Family-Wise Error Rate Controlling Procedures for Discrete Data. *Statistics in Biopharmaceutical Research*, *12*(1), 117-128. doi:10.1080/19466315.2019.1654912

#### See Also

DiscreteFWER, DBonferroni(), DHolm(), DSidak(), DHochberg()

#### Examples

```
X1 <- c(4, 2, 2, 14, 6, 9, 4, 0, 1)
X2 <- c(0, 0, 1, 3, 2, 1, 2, 2, 2)
N1 <- rep(148, 9)
N2 <- rep(132, 9)
Y1 <- N1 - X1
Y2 <- N2 - X2
df <- data.frame(X1, Y1, X2, Y2)
df
# Computation of p-values and their supports with Fisher's exact test
library(DiscreteTests) # for Fisher's exact test
test_results <- fisher_test_pv(df)</pre>
raw_pvalues <- test_results$get_pvalues()</pre>
pCDFlist <- test_results$get_pvalue_supports()</pre>
# d-Holm without critical values; using test results object
DFWER_dep_sd_fast <- discrete_FWER(test_results)</pre>
summary(DFWER_dep_sd_fast)
# d-Holm with critical values; using extracted p-values and supports
DFWER_dep_sd_crit <- discrete_FWER(raw_pvalues, pCDFlist,</pre>
                                    critical_values = TRUE)
summary(DFWER_dep_sd_crit)
# d-Bonferroni without critical values; using test results object
DFWER_dep_fast <- discrete_FWER(test_results, single_step = TRUE)</pre>
```

summary(DFWER\_dep\_fast)

# d-Bonferroni with critical values; using extracted p-values and supports DFWER\_dep\_crit <- discrete\_FWER(raw\_pvalues, pCDFlist, single\_step = TRUE,</pre>

```
critical_values = TRUE)
summary(DFWER_dep_crit)
# d-Hochberg without critical values; using test results object
DFWER_ind_su_fast <- discrete_FWER(test_results, independence = TRUE)</pre>
summary(DFWER_ind_su_fast)
# d-Hochberg with critical values; using extracted p-values and supports
DFWER_ind_su_crit <- discrete_FWER(raw_pvalues, pCDFlist,</pre>
                                    independence = TRUE,
                                    critical_values = TRUE)
summary(DFWER_ind_su_crit)
# d-Šidák without critical values; using extracted p-values and supports
DFWER_ind_fast <- discrete_FWER(raw_pvalues, pCDFlist,</pre>
                                 independence = TRUE,
                                 single_step = TRUE)
summary(DFWER_ind_fast)
# d-Šidák with critical values; using test results object
DFWER_ind_crit <- discrete_FWER(test_results, independence = TRUE,</pre>
                                 single_step = TRUE,
                                 critical_values = TRUE)
summary(DFWER_ind_crit)
```

```
DSidak
```

Discrete Šidák Procedure for Independent Tests

#### Description

DSidak() is a wrapper function of discrete\_FWER() for computing the discrete Sidák procedure for independent discrete tests. It simply passes its arguments to discrete\_FWER() with fixed independence = TRUE and single\_step = TRUE.

#### Usage

```
DSidak(test_results, ...)
## Default S3 method:
DSidak(
   test_results,
   pCDFlist,
   alpha = 0.05,
   critical_values = FALSE,
   select_threshold = 1,
   pCDFlist_indices = NULL,
   ...
)
```

### DSidak

```
## S3 method for class 'DiscreteTestResults'
DSidak(
   test_results,
   alpha = 0.05,
   critical_values = FALSE,
   select_threshold = 1,
   ...
)
```

### Arguments

test_results	either a numeric vector with <i>p</i> -values or an R6 object of class DiscreteTestResults from package DiscreteTests for which a discrete FWER procedure is to be performed.	
	further arguments to be passed to or from other methods. They are ignored here.	
pCDFlist	list of the supports of the CDFs of the <i>p</i> -values; each list item must be a numeric vector, which is sorted in increasing order and whose last element equals 1.	
alpha	single real number strictly between 0 and 1 indicating the target FWER level.	
critical_values	5	
	single boolean specifying whether critical constants are to be computed.	
select_threshold		
	single real number strictly between 0 and 1 indicating the largest raw $p$ -value to be considered, i.e. only $p$ -values below this threshold are considered and the procedures are adjusted in order to take this selection effect into account; if select_threshold = 1 (the default), all raw $p$ -values are selected.	
pCDFlist_indices		
	list of numeric vectors containing the test indices that indicate to which raw <i>p</i> -value(s) each support in pCDFlist belongs; if NULL (the default) the lengths of test_results and pCDFlist <b>must</b> be equal.	

### Details

Computing critical constants (critical\_values = TRUE) requires considerably more execution time, especially if the number of unique supports is large. We recommend that users should only have them calculated when they need them, e.g. for illustrating the rejection set in a plot or other theoretical reasons. Setting (critical\_values = FALSE) is sufficient for obtaining rejection decisions and adjusted *p*-values.

### Value

A DiscreteFWER S3 class object whose elements are:

Rejected	rejected raw p-values.
Indices	indices of rejected hypotheses.
Num_rejected	number of rejections.
Adjusted	adjusted <i>p</i> -values.

Critical_consta	ants
	critical values (only exists if computations where performed with critical_values = TRUE).
Data	list with input data.
Data\$Method	character string describing the performed algorithm, e.g. 'Discrete Bonferroni procedure'.
Data\$Raw_pvalue	25
	observed <i>p</i> -values.
Data\$pCDFlist	list of the <i>p</i> -value supports.
Data\$FWER_level	
	FWER level alpha.
Data\$Independer	
	boolean indicating whether the <i>p</i> -values were considered as independent.
Data\$Single_ste	
	boolean indicating whether a single-step or step-down procedure was performed.
Data\$Data_name	the respective variable names of the input data.
Select	list with data related to <i>p</i> -value selection; only exists if select_threshold < 1.
Select\$Threshold	
	<i>p</i> -value selection threshold (select_threshold).
Select\$Effectiv	/e_Thresholds
	results of each <i>p</i> -value CDF evaluated at the selection threshold.
Select\$Pvalues	selected <i>p</i> -values that are $\leq$ selection threshold.
Select\$Indices	indices of $p$ -values $\leq$ selection threshold.
Select\$Scaled	scaled selected <i>p</i> -values.
Select\$Number	number of selected $p$ -values $\leq$ selection threshold.

### References

Döhler, S. (2010). Validation of credit default probabilities using multiple-testing procedures. *Journal of Risk Model Validation*, 4(4), 59-92. doi:10.21314/JRMV.2010.062

### See Also

discrete\_FWER(), DHochberg(), DBonferroni(), DHolm()

### Examples

X1 <- c(4, 2, 2, 14, 6, 9, 4, 0, 1) X2 <- c(0, 0, 1, 3, 2, 1, 2, 2, 2) N1 <- rep(148, 9) N2 <- rep(132, 9) Y1 <- N1 - X1 Y2 <- N2 - X2 df <- data.frame(X1, Y1, X2, Y2) df

```
# Computation of p-values and their supports with Fisher's exact test
library(DiscreteTests) # for Fisher's exact test
test_results <- fisher_test_pv(df)
raw_pvalues <- test_results$get_pvalues()
pCDFlist <- test_results$get_pvalue_supports()
# d-Šidák without critical values; using extracted p-values and supports
DSidak_fast <- DSidak(raw_pvalues, pCDFlist)
summary(DSidak_fast)
```

```
# d-Šidák with critical values; using test results object
DSidak_crit <- DSidak(test_results, critical_values = TRUE)
summary(DSidak_crit)
```

hist.DiscreteFWER Histogram of Raw P-Values

### Description

Computes a histogram of the raw p-values of a DiscreteFWER object.

#### Usage

```
## S3 method for class 'DiscreteFWER'
hist(x, breaks = "FD", mode = c("raw", "selected"), ...)
```

#### Arguments

х	an object of class DiscreteFWER.
breaks	as in graphics::hist(); here, the Friedman-Diaconis algorithm ("FD") is used as default.
mode	single character string specifying for which \$p\$-values the histogram is to be generated; must either be "raw" or "selected".
	<pre>further arguments to graphics::hist() or graphics::plot.histogram(), respectively.</pre>

#### Details

If x does not contain results of a selection approach, a warning is issued and a histogram of the raw p-values is drawn.

#### Value

An object of class histogram.

### Examples

```
X1 <- c(4, 2, 2, 14, 6, 9, 4, 0, 1)
X2 <- c(0, 0, 1, 3, 2, 1, 2, 2, 2)
N1 <- rep(148, 9)
N2 <- rep(132, 9)
Y1 <- N1 - X1
Y2 <- N2 - X2
df <- data.frame(X1, Y1, X2, Y2)
df
# Computation of p-values and their supports with Fisher's exact test
library(DiscreteTests) # for Fisher's exact test
test_results <- fisher_test_pv(df)</pre>
raw_pvalues <- test_results$get_pvalues()</pre>
pCDFlist <- test_results$get_pvalue_supports()</pre>
# d-Holm with critical values; using test results object
DHolm_crit <- DHolm(test_results, critical_values = TRUE)</pre>
hist(DHolm_crit)
```

plot.DiscreteFWER *Plot Method for* DiscreteFWER *objects* 

#### Description

Plots raw p-values of a DiscreteFWER object and highlights rejected and accepted p-values. If calculated, the critical values are plotted, too.

### Usage

```
## S3 method for class 'DiscreteFWER'
plot(
    x,
    col = c(2, 4, 1),
    pch = c(20, 20, 17),
    lwd = rep(par()$lwd, 3),
    cex = rep(par()$cex, 3),
    type_crit = "b",
    legend = NULL,
    ...
)
```

#### Arguments

х	object of class DiscreteFWER.
col	numeric or character vector of length 3 indicating the colours of the
	1. rejected p-values

3. critical values (if present).	ers of the
	ers of the
pch numeric or character vector of length 3 indicating the point character	
1. rejected p-values	
2. accepted p-values	
3. critical values (if present and type_crit is a plot type like 'p'	', 'b' etc.).
1wdnumeric vector of length 3 indicating the thickness of the points at faults to current par()\$1wd setting.	nd lines; de-
cex numeric vector of length 3 indicating the size of point characters or	lines of the
1. rejected p-values	
2. accepted p-values	
3. critical values (if present).	
defaults to current par()\$cex setting.	
type_crit 1-character string giving the type of plot desired for the critical value '1' etc; see plot()).	es (e.g.: 'p',
legendif NULL, no legend is plotted; otherwise expecting a character string lietc. or a numeric vector of two elements indicating (x, y) coordinate	
further arguments to plot.default().	

#### Value

A plot is created, but no value is returned.

### Examples

```
X1 <- c(4, 2, 2, 14, 6, 9, 4, 0, 1)
X2 <- c(0, 0, 1, 3, 2, 1, 2, 2, 2)
N1 <- rep(148, 9)
N2 <- rep(132, 9)
Y1 <- N1 - X1
Y2 <- N2 - X2
df <- data.frame(X1, Y1, X2, Y2)
df
# Computation of p-values and their supports with Fisher's exact test
library(DiscreteTests) # for Fisher's exact test
test_results <- fisher_test_pv(df)</pre>
raw_pvalues <- test_results$get_pvalues()</pre>
pCDFlist <- test_results$get_pvalue_supports()</pre>
DBonf_fast <- DBonferroni(raw_pvalues, pCDFlist)</pre>
DBonf_crit <- DBonferroni(test_results, critical_values = TRUE)</pre>
DHolm_fast <- DHolm(test_results)</pre>
DHolm_crit <- DHolm(raw_pvalues, pCDFlist, critical_values = TRUE)</pre>
plot(DBonf_fast)
plot(DBonf_crit, xlim = c(1, 5), ylim = c(0, 0.4))
```

```
plot(DHolm_fast, col = c(2, 4), pch = c(2, 3), lwd = c(2, 2),
    legend = "topleft", xlim = c(1, 5), ylim = c(0, 0.4))
plot(DHolm_crit, col = c(2, 4, 1), pch = c(1, 1, 4), lwd = c(1, 1, 2),
    type_crit = 'o', legend = c(1, 0.4), lty = 1, xlim = c(1, 5),
    ylim = c(0, 0.4))
```

print.DiscreteFWER Printing discrete FWER results

### Description

Prints the results of discrete FWER analysis, stored in a DiscreteFWER S3 class object.

### Usage

## S3 method for class 'DiscreteFWER'
print(x, ...)

#### Arguments

х	object of class DiscreteFWER.
	further arguments to be passed to or from other methods. They are ignored in this function.

#### Value

The respective input object is invisibly returned via invisible(x).

#### Examples

```
X1 <- c(4, 2, 2, 14, 6, 9, 4, 0, 1)
X2 <- c(0, 0, 1, 3, 2, 1, 2, 2, 2)
N1 <- rep(148, 9)
N2 <- rep(132, 9)
Y1 <- N1 - X1
Y2 <- N2 - X2
df <- data.frame(X1, Y1, X2, Y2)
df
# Computation of p-values and their supports with Fisher's exact test
library(DiscreteTests) # for Fisher's exact test
test_results <- fisher_test_pv(df)</pre>
raw_pvalues <- test_results$get_pvalues()</pre>
pCDFlist <- test_results$get_pvalue_supports()</pre>
# d-Holm with critical values; using test results object
DHolm_crit <- DHolm(test_results, critical.values = TRUE)</pre>
# print results
print(DHolm_crit)
```

#### Description

summary method for class DiscreteFWER.

#### Usage

```
## S3 method for class 'DiscreteFWER'
summary(object, ...)
## S3 method for class 'summary.DiscreteFWER'
print(x, max = NULL, ...)
```

#### Arguments

object	an object of class DiscreteFWER.
	further arguments passed to or from other methods.
x	an object of class summary.DiscreteFWER.
max	numeric or NULL, specifying the maximal number of <i>rows</i> of the p-value table to be printed. By default, when NULL, getOption("max.print") is used.

### Details

summary.DiscreteFWER objects contain all data of an DiscreteFWER object, but also include an additional table which includes the raw p-values, their indices, the respective critical values (if present), the adjusted p-values (if present) and a logical column to indicate rejection. The table is sorted in ascending order by the raw p-values.

print.summary.DiscreteFWER simply prints the same output as print.DiscreteFWER, but also prints the p-value table.

#### Value

summary.DiscreteFWER computes and returns a list that includes all the data of an input DiscreteFWER object, plus

Tabledata.frame, sorted by the raw p-values, that contains the indices, the raw p-<br/>values themselves, their respective critical values (if present), their adjusted p-<br/>values (if present) and a logical column to indicate rejection.

### Examples

```
X1 <- c(4, 2, 2, 14, 6, 9, 4, 0, 1)
X2 <- c(0, 0, 1, 3, 2, 1, 2, 2, 2)
N1 <- rep(148, 9)
N2 <- rep(132, 9)
Y1 <- N1 - X1
Y2 <- N2 - X2
df <- data.frame(X1, Y1, X2, Y2)
df
# Computation of p-values and their supports with Fisher's exact test
library(DiscreteTests) # for Fisher's exact test
test_results <- fisher_test_pv(df)</pre>
raw_pvalues <- test_results$get_pvalues()</pre>
pCDFlist <- test_results$get_pvalue_supports()</pre>
# d-Holm procedure without critical values; using test results object
DFWER_dep_sd_fast <- discrete_FWER(test_results)</pre>
summary(DFWER_dep_sd_fast)
# d-Bonferroni procedure with critical values; using test results object
DFWER_dep_crit <- discrete_FWER(test_results, single_step = TRUE,</pre>
critical_values = TRUE)
summary(DFWER_dep_crit)
```

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