

# Package ‘fertilmodel’

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**Type** Package

**Title** Fertility Models

**Version** 1.4

**Date** 2024-11-29

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**Maintainer** Michail Tsagris <mtsagris@uoc.gr>

**Depends** R (>= 4.0)

**Imports** quadprog, stats

**Suggests** Rfast, Rfast2

**Description** Four fertility models are fitted using non-linear least squares. These are the Hadwiger, the Gamma, the Model1 and Model2, following the terminology of the following paper: Peristera P. and Kostaki A. (2007). ``Modeling fertility in modern populations''. Demographic Research, 16(6): 141--194. <doi:10.4054/DemRes.2007.16.6>. Model based averaging is also supported.

**License** GPL (>= 2)

**NeedsCompilation** no

**Repository** CRAN

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fertilmodel-package    *This is an R package that fits 4 fertility models.*

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### Description

Four fertility models are fitted using non-linear least squares. These are the Hadwiger, the Gamma, the Model1 and Model2, following the terminology of the following paper: Peristera P. and Kostaki A. (2007). Modeling fertility in modern populations. *Demographic Research*, 16(6): 141–194. <doi:10.4054/DemRes.2007.16.6>.

### Details

Package: nlgmcr Type: Package  
Version: 1.4  
Date: 2024-11-29  
License: GPL-2

### Maintainers

Michail Tsagris <mtsagris@uoc.gr>.

### Note

**Acknowledgments:** This package is dedicated to Sanaa who introduced me to these models and whom I may never see again.

### Author(s)

Michail Tsagris <mtsagris@uoc.gr>.

### References

Peristera P. and Kostaki A. (2007). Modeling fertility in modern populations. *Demographic Research*, 16(6), 141–194. <doi:10.4054/DemRes.2007.16.6>.

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comb	<i>Model based average of the estimated values from two or more fertility models</i>
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**Description**

Model based average of the estimated values from two or more fertility models.

**Usage**

```
comb(models)
```

**Arguments**

models            A list with possible models.

**Value**

A list including:

weights            The weights assigned to each model.

fit                The weighted fitted age-specific fertility rates  $\hat{f}(x)$ .

**Author(s)**

Michail Tsagris.

R implementation and documentation: Michail Tsagris <mtsagris@uoc.gr>.

**See Also**

[Hadwiger](#)

**Examples**

```
rate <- c(0.0001, 0.0006, 0.0033, 0.0111, 0.0263, 0.0412, 0.0544, 0.0622,  
0.0660, 0.0704,0.0723, 0.0753, 0.0814, 0.0873, 0.0924, 0.0962, 0.0989,  
0.1006, 0.0990, 0.0933,0.0831, 0.0747, 0.0634, 0.0529, 0.0424, 0.0326,  
0.0242, 0.0172, 0.0115, 0.0073, 0.0040, 0.0022, 0.0012, 0.0006, 0.0003,  
0.0002, 0.0001)  
age <- 13:49  
mod1 <- Hadwiger(rate, age)  
mod2 <- Gama(rate, age)  
mod3 <- Model1(rate, age)  
mod4 <- Model2(rate, age)  
a <- list(mod1 = mod1, mod2 = mod2, mod3 = mod3, mod4 = mod4)  
comb(a)
```

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fertil.plot	<i>Plot of the age-specific fertility rates and the estimated values from one or more fertility models</i>
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### Description

Plot of the age-specific fertility rates and the estimated values from one or more fertility models.

### Usage

```
fertil.plot(rate, age, fit = NULL, grid = FALSE, names = NULL)
```

### Arguments

rate	A vector with the age-specific fertility rates.
age	A vector with the age of the women.
fit	Here you can specify nothing (only plot the fertility rates across the ages), or you can specify a vector or a matrix with fitted values from at least one model.
grid	Do you want a grid of vertical and horizontal lines? TRUE or FALSE.
names	If you provided fitted models from a model, you can specify the name(s) of the model(s) so that they appear as a legend.

### Value

A plot with the age-specific fertility rates across the mothers' age and perhaps the fitted values from at least one model.

### Author(s)

Michail Tsagris.

R implementation and documentation: Michail Tsagris <mtsagris@uoc.gr>.

### See Also

[Hadwiger](#), [comb](#)

### Examples

```
rate <- c(0.0000, 0.0005, 0.0023, 0.0073, 0.0175, 0.0283,  
0.0420, 0.0523, 0.0601, 0.0712, 0.0789, 0.0865, 0.0939,  
0.0991, 0.1052, 0.1115, 0.1155, 0.1178, 0.1211, 0.1159,  
0.1104, 0.1031, 0.0916, 0.0776, 0.0639, 0.0498, 0.0387,  
0.0281, 0.0185, 0.0117, 0.0067, 0.0038, 0.0020, 0.0011,  
0.0007, 0.0003, 0.0002)  
age <- 13:49  
mod1 <- Hadwiger(rate, age)  
mod2 <- Gama(rate, age)
```

```
fertil.plot(rate, age)
fertil.plot(rate, age, cbind(mod1$fit, mod2$fit), grid = TRUE, names = c("Hadwiger", "Gama"))
```

Hadwiger

*Fertility models***Description**

Fertility models.

**Usage**

```
Hadwiger(rate, age)
Gama(rate, age)
Model1(rate, age)
Model2(rate, age)
```

**Arguments**

`rate`            A vector with the age-specific fertility rates.  
`age`             A vector with the age of the women.

**Details**

The following fertility models are fitted: Hadwiger:

$$f(x) = \frac{ab}{c} \left(\frac{c}{x}\right)^{3/2} \exp\left[-b^2\left(\frac{c}{x} + \frac{x}{c} - 2\right)\right],$$

where  $x$  is the age of the mother at birth,  $a$  is associated with total fertility, the parameter  $b$  determines the height of the curve and the parameter  $c$  is related to the mean age of motherhood.

Gama:

$$f(x) = R \frac{1}{\Gamma(b)c^b} (x-d)^{b-1} \exp\left(-\frac{x-d}{c}\right),$$

where  $d$  represents the lower age at childbearing, while the parameter  $R$  determines the level of fertility.

Model1:

$$f(x) = c_1 \exp\left[-\frac{(x-\mu)^2}{\sigma^2(x)}\right],$$

where  $\sigma(x) = \sigma_{11}$  if  $x \leq \mu$  and  $\sigma(x) = \sigma_{12}$  if  $x > \mu$ . The parameter  $c_1$  describes the base level of the fertility curve and is associated with the total fertility rate,  $\mu$  reflects the location of the distribution, i.e. the modal age and  $\sigma_{11}$  and  $\sigma_{12}$  reflect the spread of the distribution before and after its peak, respectively.

Model2:

$$f(x) = c_1 \exp\left[-\frac{(x-\mu_1)^2}{\sigma_1^2}\right] + c_2 \exp\left[-\frac{(x-\mu_2)^2}{\sigma_2^2}\right],$$

where the parameters  $c_1$  and  $c_2$  express the severity i.e. the total fertility rates of the first and the second hump respectively,  $\mu_1$  and  $\mu_2$  are related to the mean ages of the two subpopulations the one with earlier fertility and the other with fertility at later ages, while  $\sigma_1$  and  $\sigma_2$  reflect the variances of the two humps.

### Value

A list including:

param	The vector of the estimated parameters.
sse	The sum of squares of the errors $\sum_{i=1}^n (f_x - \hat{f}(x))^2$ , where $f_x$ denotes the observed age-specific fertility rates and $\hat{f}(x)$ denote the fitted age-specific fertility rates.
fx	The fitted values, the fitted age-specific fertility rates $\hat{f}(x)$ .
res	The residuals, $f_x - \hat{f}_x$ .

### Author(s)

Michail Tsagris.

R implementation and documentation: Michail Tsagris <mtsagris@uoc.gr>.

### References

Peristera P. and Kostaki A. (2007). Modeling fertility in modern populations. *Demographic Research*, 16(6): 141–194.

### See Also

[fertil.plot](#), [comb](#)

### Examples

```
rate <- c(0.0001, 0.0006, 0.0033, 0.0111, 0.0263, 0.0412, 0.0544, 0.0622,
0.0660, 0.0704,0.0723, 0.0753, 0.0814, 0.0873, 0.0924, 0.0962, 0.0989,
0.1006, 0.0990, 0.0933,0.0831, 0.0747, 0.0634, 0.0529, 0.0424, 0.0326,
0.0242, 0.0172, 0.0115, 0.0073, 0.0040, 0.0022, 0.0012, 0.0006, 0.0003,
0.0002, 0.0001)
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```

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